# Reflective Antenna Arrays Based on Shorted Ring Slots

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***Abstract* — Reflective arrays based on shorted ring slots are analyzed. It is proven that reflection angle for the normally incident circularly polarized wave can be effectively changed by proper situation of the conductive shorts. Thus, phased arrays with wide-angle scanning can be built using arrays of shorted ring slots.**

## I. INTRODUCTION

Development of the light and low-cost Ka-band phased arrays is of primary importance due to the growing interest in the Ka-band satellite communications [1]. One of the main obstacles that delays the design of the millimeter-wave phased arrays is the absence of fast low-loss phase shifters. To overcome this limitation, low-loss Ka-band polarization phase shifters, based on circular waveguide, have been proposed [2,3]. Unfortunately, reflective phased arrays based on circular waveguides are characterized by a reduced scanning sector because of the cut-off properties of the circular waveguide.

In this work we analyze the possibility of using open polarization-type phase shifter based on a shorted ring slot in order to obtain a large sector of scanning. The first array using a similar principle (Spyraphase) was proposed by Phelan and was based on half-wave dipoles [4].

## II. PRINCIPLE OF OPERATION

Arrays of ring slots have been widely investigated as frequency selective surfaces [5].

The main property of these arrays is the resonant behavior of the reflection coefficient. The resonance occurs when the circumference of the ring slot is approximately equal to the wavelength $\lambda$. At the resonant frequency $\omega_r$, the periodic array based on ring slots is transparent to the incident plane wave.

In this work, we analyze an array that contains shorted ring slots situated at the nodes of a periodic rectangular grid. The presence of the uniformly situated metal shorts does not affect the scattering of the plane wave with the plane of polarization orthogonal to the metal shorts. So, the array is still transparent to this wave at the resonant frequency $\omega_r$. However, at the same frequency $\omega_r$, the plane wave with the polarization plane parallel to the metal shorts will be mainly reflected by the array due to the induced electric currents that flow across the metal shorts.

Now assume that the conductive plane is situated at the distance $\lambda/4$ from the array. The no-load condition with reflection coefficient of 1 is obtained in the plane of the ring slots for the wave with polarization plane orthogonal to the metal shorts. The wave with polarization plane parallel to the metal shorts is reflected from the array with the reflection coefficient of –1. Therefore, a differential phase shift of 180 degrees appears between two reflected waves with orthogonal polarizations.

According to Fox's principle of phase changing [6], the reflection of a circularly polarized wave from the array with uniform angular position $\gamma$ of shorts leads to the appearance of the additional phase shift of $2\gamma$ in the reflected circularly polarized wave.

However, non-uniform position of the shorts (Fig. 1) results in the presence of a non-uniform phase shift across the reflected circularly polarized wave. It is possible to introduce a linearly distributed phase shift across the reflected plane wave and to redirect it as desired. Therefore, with proper positioning of metal shorts, the angle of reflection can be changed.

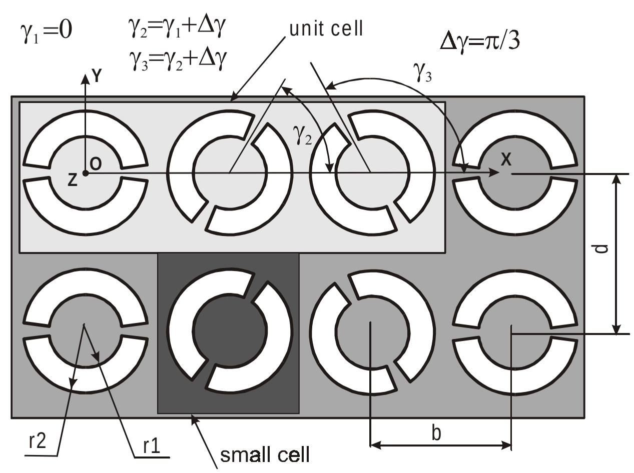


Fig. 1. Reflective array based on shorted ring slots. Configuration of the array when *N*=3.

## III. Mathematical Model

Strong mutual coupling is the main characteristic of phased arrays based on open phase shifters. To take into account this effect, a mathematical model has been built.

We consider an infinite reflective array of shorted ring slots arranged in a regular rectangular grid. A small cell of this array contains only one shorted ring slot.

Now we assume the scanning in the plane XOZ. The normally incident circularly polarized wave moving towards the array must be reflected in the direction determined by the elevation angle $\theta$. To do this, two small adjacent cells must provide an incremental phase difference of $\psi_{xm}$ to the reflected wave. This phase difference can be calculated as follows:

$$\psi_{xm} = -k\,b\sin\theta\,, \tag{1}$$

where $k$ is the wavenumber and $b$ is *x*-size of the small cell.

According to the polarization principle of phase changing, the difference between angular positions of shorts in two adjacent cells $\Delta\gamma$ can be set to:

$$\Delta\gamma = \psi_{xm}/2 = -kb\sin\theta/2 \tag{2}$$

Non-uniform angular positions of the shorts destroy the periodicity of the array. Therefore, the small cell cannot be considered as the unit cell of the array. However, if $\Delta\gamma$ can be presented as:

$$\Delta\gamma = \pi M/N \ \ (N = 1,2...,M = 0,1,...N-1), \tag{3}$$

the reflective array is converted in a periodic structure. The unit cell of this array contains $N$ adjacent small cells arranged in the x-direction. In fact, any $\Delta\gamma$ can be approximated with sufficient tolerance by (3). Therefore, the infinite reflective array can be analyzed as a periodic structure using the Floquet theorem [7].

According to the Floquet theorem, the electromagnetic field above the reflective array is presented as a sum of Floquet modes or plane waves propagating in certain directions. Condition (3) assures that one of the Floquet modes is the plane wave propagating in the desired direction determined by the angle $\theta$. Therefore, this reflective array converts the incident Floquet mode into the desired Floquet mode.

To calculate the efficiency of this conversion, a system of integral equations was formulated. The unknown tangential electric field in the plane of the ring slots $\vec{E}_\tau$ was presented as a sum of two components:

$$\vec{E}_\tau = \vec{E}_\tau^u + \vec{E}_\tau^I\,, \tag{4}$$

where $\vec{E}_\tau^u$ is the electric field that appears when the incident wave excites the reflective array with "unshorted" ring slots and $\vec{E}_\tau^I$ is the electric field that exists due to the electric currents $I_m$ that flow across the metal shorts.

The traditional approach [5] was used to calculate the electric field $\vec{E}_\tau^u$. As a result, the electric field $\vec{E}_{\tau n}^u$ in the ring slot number $n$ is presented in the following form:

$$\vec{E}_{\tau n}^u = \sum_{l=1}^{L} D_{nl}\vec{W}_l\,, \tag{5}$$

where $\vec{W}$ is the system of vector functions of the coaxial waveguide and $D_{nl}$ are the corresponding magnitudes.

The integral equation with respect to the unknown electric field $\vec{E}_\tau^I$ was obtained as a result of the application of the continuity condition for the tangential magnetic field across all shorted ring slots contained in the unit cell.

An important consideration is that the magnetic field is not continuous in the regions where electric currents flow across the metal shorts. Thus, the following integral equation is obtained:

$$\int_S \left( \sum_{n=1}^{\infty} \left(y_n + y_n^m\right) \vec{\Psi}_n(x,y) \cdot \vec{\Psi}_n^*(x',y') \right) \vec{E}_\tau^I(x',y')\,dS' = -\sum_{m=1}^{2N} I_m \vec{F}_m(x,y), \tag{6}$$

where $\vec{\Psi}_n$ is the orthogonal system of vector normalized Floquet modes corresponding to the unit cell; $y_n$ is the wave admittance of the Floquet mode $\vec{\Psi}_n$; $y_n^m$ is the modified admittance of $\vec{\Psi}_n$ that takes into account the presence of the metal screen and the properties of the dielectric substrate; $S$ is the total area of all ring slots contained in the unit cell and $\vec{F}_m$ is the vector function that converts electric current $I_m$ to the corresponding electric current density $\vec{J}_m$.

Equation (6) is valid only on the surface of all ring slots that belong to the unit cell. This equation was solved using the Galerkin method. Vector functions $\vec{W}_l$ of the coaxial waveguide were used as the basis and weighting functions. As a result the electric field $\vec{E}_{\tau n}^I$ in the ring slot number $n$ was presented in the following form:

$$\vec{E}_{\tau n}^I = \sum_{l=1}^{L} C_{nl}\vec{W}_l\,, \tag{7}$$

where $C_{nl}$ are the magnitude coefficients that can be expressed as linear functions of unknown currents $I_m$.

Electric fields $\vec{E}_\tau^u$ and $\vec{E}_\tau^I$ satisfy boundary conditions all over the unit cell with the exception of regions of shorts where electric currents $I_m$ flow. Application of the boundary condition for the total electric field $\vec{E}_\tau$ at each metal short gives the additional *2N* equations. Solution of these equations permits us to calculate all currents $I_m$. Therefore, the electric field $\vec{E}_\tau$ in the plane of ring slots can be calculated and the electric field above the reflect array can be presented as follows:

$$\vec{E}_a = A_1\left(\vec{\Psi}_1 + j\vec{\Psi}_2\right)e^{jkz} + \sum_{m=1}^{\infty} B_m \vec{\Psi}_m e^{-j\beta_m z}, \quad (8)$$

where $\beta_m$ is the propagation constant of the corresponding Floquet mode, $A_1$ is the magnitude of the normally incident wave and $B_m$ are the magnitudes of the reflected waves.

The first component of (8) is the normally incident circularly polarized wave propagating towards the reflective array, while the infinite sum represents the reflected plane waves.

TABLE I
SCATTERING OF THE NORMALLY INCIDENT CIRCULARLY POLARIZED WAVE (F=31GHZ)

| reflection angle $\theta$ (degrees) | $\Delta\gamma$, degrees | N | M | Conversion loss L,(dB) | Axial Ratio (reflected wave) (dB) |
|---|---|---|---|---|---|
| 9.28 | 15 | 12 | 1 | 0.18 | 0.045 |
| 10.1 | 16.4 | 11 | 1 | 0.18 | 0.058 |
| 11.2 | 18 | 10 | 1 | 0.19 | 0.19 |
| 12.4 | 20 | 9 | 1 | 0.18 | 0.075 |
| 14 | 22.5 | 8 | 1 | 0.17 | 0.07 |
| 16.05 | 25.7 | 7 | 1 | 0.18 | 0.1 |
| 18.8 | 30 | 6 | 1 | 0.18 | 0.1 |
| 20.6 | 32.7 | 11 | 2 | 0.19 | 0.36 |
| 22.8 | 36 | 5 | 1 | 0.2 | 0.39 |
| 25.5 | 40 | 9 | 2 | 0.21 | 0.11 |
| 28.9 | 45 | 4 | 1 | 0.21 | 0.15 |
| 31.9 | 49.1 | 11 | 3 | 0.23 | 0.49 |
| 33.6 | 51.4 | 7 | 2 | 0.29 | 0.84 |
| 35.5 | 54 | 10 | 3 | 0.33 | 1.55 |
| 40.2 | 60 | 3 | 1 | 0.29 | 1.13 |
| 44.7 | 65.5 | 11 | 4 | 0.45 | 0.91 |
| 46.5 | 67.5 | 8 | 3 | 0.85 | 2.96 |
| 50.7 | 72 | 5 | 2 | 0.86 | 6.39 |
| 53.7 | 75 | 12 | 5 | 1.93 | 11.35 |
| 56 | 77.1 | 7 | 3 | 1.49 | 8.06 |
| 59.3 | 80 | 9 | 4 | 1.54 | 7.75 |
| 61.6 | 81.8 | 11 | 5 | 1.25 | 6.13 |
| 63.3 | 83.1 | 13 | 6 | 1.44 | 6.43 |
| 64.8 | 84 | 15 | 7 | 1.70 | 7.16 |
| 65.6 | 84.7 | 17 | 8 | 1.89 | 7.74 |

Two of the reflected Floquet modes (one TE and one TM mode) propagate in the desired direction determined by the angle $\theta$. Therefore, the conversion loss $L$ can be calculated as a ratio between the power density of the "desired" modes and the power density of the incident modes:

$$L = \frac{y_{m1}\left|B_{m1}\right|^2 + y_{m2}\left|B_{m2}\right|^2}{2y_1\left|A_1\right|^2}, \quad (9)$$

where $m1$ and $m2$ are index numbers corresponding to the "desired" Floquet modes – plane waves propagating towards $\theta$.

## III. RESULTS OF SIMULATION

Scattering of the incident circularly polarized wave on the reflective array with b=5mm, d=5mm, r1=1.65mm, r2=2.17mm and the distance of 2.8mm between the screen and the plane of shorted ring slots was simulated. It was assumed that ring slots were printed on the substrate with a relative dielectric permitivity of 1.

The results of the scattering of the normally incident right-hand circularly polarized wave are given in Table I. The information concerning polarization of the reflected "desired" wave is presented in the last column of Table I.

According to the results presented in Table I, the reflective array demonstrates excellent performance when the angle of reflection $\theta$ is less than 45 degrees. Conversion loss for this case is less than 0.5 dB and polarization of the reflected wave is close to circular. Further increasing of the angle $\theta$ leads to the significant distortions in the polarization of the reflected wave. However, conversion loss is less than 2dB when the reflection angle $\theta$ is less than 65 degrees.

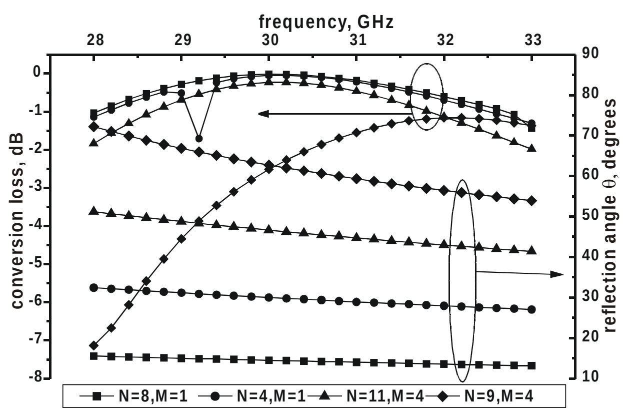


Fig. 2. Frequency dependence of the conversion loss *L* and the reflection angle for 4 configurations of the reflective array.

Dependence of the conversion loss with respect to frequency is demonstrated in fig. 2. Conversion loss for 4 different configurations of the reflective array is presented. These configurations correspond to the reflection angles when $\theta$ is $15^0$, $30^0$, $45^0$ and $60^0$ at the frequency of 31GHz. It should be taken into account that the change of frequency leads to a change in the reflection angle if the configuration of the reflective array remains constant. Thus, corresponding reflection angles $\theta$ are also given in fig. 2. According to fig. 2, the reflective array demonstrates good performance in the analyzed frequency range. However, an abnormal point can be observed when $N$=4 and $M$=1 at the frequency of 29.2GHz. Detailed study reveals that in this case, a vertical component of the incident wave is almost entirely converted to the reflected waves with reflection angles of $\theta = 0$, $\theta = 31^0$ and $\theta = -31^0$.

## IV. Experimental Verification

The method of waveguide simulator [7] was used to verify the mathematical model developed. A metal diaphragm that contained two shorted ring slots (Fig. 3) was printed on a dielectric substrate with relative permitivity of 3.40 and a thickness of 0.51mm. The angle between the horizontal axis and the axis that determines the angular position of the metal shorts was $45^0$. The inner and the outer radiuses of the ring slots were 3.78mm and 4.47mm, respectively. This diaphragm was then installed in the cross section of the rectangular waveguide WR-90. The scattering of the $TE_{10}$ waveguide mode on this diaphragm is equivalent [7] to the scattering of the corresponding $TE$ Floquet mode on the array of shorted ring slots shown in Fig. 3.

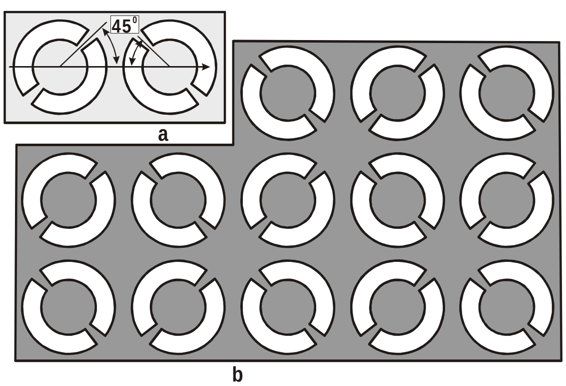


Fig. 3. (a) Cross-section of the waveguide simulator. (b) Corresponding array of shorted ring slots. Dimensions of the small cell *b* and *d* are 11.4mm and 10.2mm, respectively

The comparison between the measured reflection coefficient in the case of the waveguide simulator and the calculated reflection coefficient in the case of the scattering of the plane wave on the array of shorted ring slots is given in fig. 4. Close coincidence between measured and calculated results is observed.

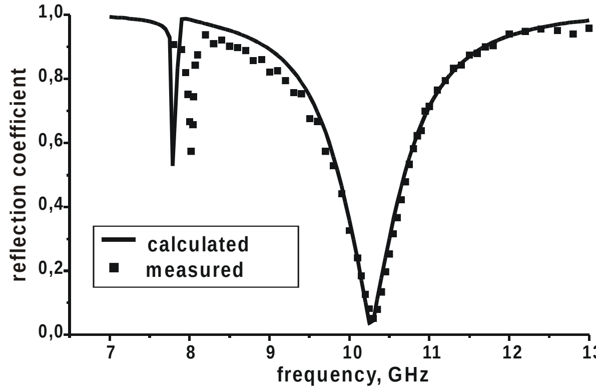


Fig. 4. Comparison between measured reflection coefficient in the case of the waveguide simulator and calculated reflection coefficient in the case of the respective array of shorted rings

## V. Conclusions

Effective control of the reflection angle for the normally incident circularly polarized wave can be achieved with the reflective array based on shorted ring slots by proper situation of the conductive shorts. It was shown that the incident wave can be effectively redirected in the directions determined by elevation angles as high as 65 degrees. Excellent redirection with small conversion loss and with conservation of the polarization of the incident wave can be achieved for elevation angles up to 45 degrees. The usage of the p-i-n diodes instead of metal shorts leads to the design of lightweight plane phased arrays with wide-angle scanning.

## Acknowledgment

This work was partially supported by the UNAM PAPIIT project IN111999.

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